Let's calculate primes! Part III - Intersection of times tables: sum formula

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Sat 13 Jan 2018 in Math, Carolin Zöbelein

In the post before, we looked at the intersection ot times tables and its problems. Now we will look at the belonging sum formula, which is given by

$$\sum_{x_i=l'}^{u'} \sum_{x_j=l}^{u} 10^{(2x_i+1)x_j+x_i}$$

and with our results before as

$$\sum_{x_i=l'}^{u'} \left(-\frac{10^{(2x_i+1)l+x_i} - 10^{(2x_i+1)(u+1)+x_i}}{10^{2x_i+1}-1} \right)$$

.

We are not able to solve this second sum analytical, but it can be interesting to look a bit around this sum. I had a look what wolframalpha.com is able to tell me about our first sum result and I found one thing which catched my attention.

$$\int -\frac{10^x \left(10^{l(2x+1)} - 10^{(u+1)(2x+1)}\right)}{10^{2x+1}-1} \mathrm{d}x$$

$$= \frac{10^x \left(\frac{10^{2lx+l} {}_2F_1\left(1,l+\frac{1}{2},l+\frac{3}{2},10^{2x+1}\right)}{2l+1} - \frac{10^{(u+1)(2x+1)} {}_2F_1\left(1,u+\frac{3}{2},u+\frac{5}{2},10^{2x+1}\right)}{2u+3} \right)}{\log(10)} + constant,$$

with

$${}_2F_1\left(a,b;c;x\right)$$

is the hypergeometric function.

It's deeply interesting that we are able to find an integral which is connected to the Gamma-Function over the hypergeometric function and hence to primes. Althougth we are not able to do both sums exactly, this gives us an hint, that we are maybe able to do the first calculation with a sum and the second calculation with the help of an integration.

At the moment, we can't use this information, but we should keep this in mind.