Cyclotomic polynomials for primes: Appendix

https://blog.carolin-zoebelein.de/2018/07/cyclotomic-polynomials-for-primes-appendix.html
Mon 30 Jul 2018 in Math, Carolin Zöbelein

Ok, last time, we had

$$\begin{aligned} \phi_p\left(x\right) &= \sum_{k=0}^{p-1} x^k \\ &= \sum_{k=0}^{p-1} e^{k\ln(x)} \\ &= e^{0\ln(x)} + \sum_{k=1}^{p-1} e^{k\ln(x)} \\ &= 1 + \frac{e^{\ln(x)}\left(e^{(p-1)\ln(x)} - 1\right)}{e^{\ln(x)} - 1} \end{aligned} \tag{1}$$

. All what I want to add in this small appendix is, that we can, of course, also write here

$$\begin{aligned} \phi_p\left(x\right) &= 1 + \frac{e^{\ln(x)}\left(e^{(p-1)\ln(x)} - 1\right)}{e^{\ln(x)} - 1} \\ &= 1 + \frac{x\left(x^{p-1} - 1\right)}{x - 1} \\ &= 1 + \frac{x^p - x}{x - 1} \end{aligned} \tag{2}$$

Now, if we solve this for p

$$\begin{aligned} \phi_p\left(x\right) &= 1 + \frac{x^p - x}{x - 1} \\ \phi_p\left(x\right) - 1 &= \frac{x^p - x}{x - 1} \\ \left(\phi_p\left(x\right) - 1\right)\left(x - 1\right) &= x^p - x \\ \left(\phi_p\left(x\right) - 1\right)\left(x - 1\right) + x &= x^p \\ p &= \ln\left(\frac{\left(\phi_p\left(x\right) - 1\right)\left(x - 1\right) + x}{x}\right). \end{aligned} \tag{3}$$

That’s it.